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Fixed Point Theorem for Expansion Mapping in Cone Metric space

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Abstract: **The main aim of this paper is to established common fixed point theorem for expansion mapping in complete Cone Metric space.**

Keywords: **Fixed point, cone metric space, Expansion Mapping.**

I. INTRODUCTION

In 2007 , Huang and Zhang [10] introduced the cone metric space by replacing real numbers with an ordering banch space .Many authors studied &proved some fixed point theorems[See(1,2,3,4,5,6,7,8,12,14,15)and references there in]. In 1976, Rosenholtz [14] discussed local Expansion as , f is a local expansion if every point in x has a neighborhood B on which f is expansion. After this a number of fixed point theorem for expansion mapping have been proved by park & wang,Li, Gao & Iseki , khan et al [18] park & Rhoades &Taniguchi etc[11,17] . Actually the above mentioned theorems appear to be the generalization for expansion mapping of banach contraction principle.

II. PRELIMINARIES

Definition 2.1: Let B be a real banach space & P be a subset of B . P is called a cone

if

i> P is a closed , non empty & P $\neq$\{o\}

ii> $a, b \epsilon R , a, b \geq 0 \,\&\, x , y \epsilon p$ $implies$ $ax + by \epsilon p$

iii> $x \epsilon P \,\&\, - x \epsilon P$ imply $x = 0$

Given a cone $P \subseteq B$, we define a partial Ordering "$\leq$" in B by $x \leq y$ if $y - x \epsilon P$, we write $x < y$ to denote $x \leq y$ but $x \neq y$ and $x << y$ to denote y – $x \epsilon p^0$, p^0 stands for the interior of P

Proposition2.2[5]: - Let P be a cone in a real banach space B ,If for a ϵ P , and a $\leq$ ka,For some k ϵ (0 , 1) then a = 0

Proposition2. 3[5]: - Let P be a cone in a real banach space B, If for a ϵ B & a << c , for all c ϵ p^0 than a = 0

Definition2.4: - Let X be a non empty set suppose the mapping $d: X \times X \rightarrow B$ satisfies

$0 \leq d (x , y), for\ all\ x, y \epsilon X \,\&\, d(x, y) = 0\ iff\ x = y d(x, y) = d (y, x) forall\ x, y \epsilon X d(x, y) \leq d(x, y) + d (z , y)\ forall\ x, y, z \epsilon X$.

Then d is called a cone metric on X and (X, d) is called a cone metric space .

Example 2.5[2]: Let E = R^2 , P = $\{ (x, y) \epsilon E : x, y \geq 0\} \subset R^2$,

$X = R^2$ and $d : X \times X \rightarrow E$ defined by

$d(x, y) = d\{ (x_1 , x_2), (y_1\ y_2) \} = (max \{ |x_1 - y_1 |, |x_2 - y_2 | \}, \alpha \max \{ |x_1 - y_1 |, |(x_2 - y_2 | \}, where\ \alpha \geq 0\ is\ a\ constant$ then (X , d) is a cone metric space .

Definition 2.6[3]:- Let (X, d) be a cone metric space , Let $\{ x_n \}_{n \epsilon N}$ be a sequence in X and $x \epsilon X$. If for any c ϵ E With 0 << c, there is $n_0 \epsilon$ N , s.t for all n> n_0 $d(x_n, x)$ << c. Then $\{ x_n \}_{n \in N}$ is said to be convergent to x , and x is the limit of $\{ x_n \}_{n \in N}$ we denote this by $\lim_{n\to\infty} d(x_n, x) = 0$

Definition2.7[3]: Let (X, d) be a cone metric space & $\{ x_n \}_{n \epsilon N}$ be a sequence in X If for any c ϵ E With 0 << c , there is $n_0 \epsilon$ N such that for all $m, n > n_0$, $d(x_n, x_m) << c$ then $\{ x_n \}_{n \epsilon N}$ is called a Cauchy sequence in x. we denote this by $\lim_{m,n\to\infty} d(x_n, x_m) = 0$

Definition2.8 [3]: Let (X, d) be a cone metric space & $\{ x_n \}_{n \epsilon N}$ be a sequence in X. if$\{ x_n \}_{n \epsilon N}$ is convergent, then it is a Cauchy sequence.

Definition2.9[3] : Let (X , d) be a cone metric space, if every Cauchy sequence is convergent in X ,then X is called a complete cone metric space.

Definition2.10 [3] : Let (X, d) be a cone metric space. Let T be a self map on X. If for all sequence $\{ x_n \}_{n \epsilon N}$ in X $\lim_{n\to\infty} x_n \to x \implies \lim_{n\to\infty} Tx_n \to Tx$ then T is called continuous on X.

Lemma2.11: Let (X, d) be a cone metric space. If $\{ x_n \}$ is a convergent sequence in X, then the limit of $\{ x_n \}$ is unique.

Lemma2.12: Let (X , d) be a cone metric space,$\{ x_n \}$ be a sequence in X. If $\{ x_n \}$ converges to x and $\{x_{n_k}\}$ is any subsequence of $\{ x_n \}$ then $\{x_{n_k}\}$ converges to x .

III. MAIN RESULT

Theorem: Let (X, d) be a cone metric space with respect to a cone P.Let S & T be a continuous self map satisfying .

$$d(Sx,Ty) + \alpha d(x,y) \geq \frac{\beta\, d(y,Ty)[1 + d(x,Sx)]}{1 + d(x,y)} + \gamma \max\{d(x,Sx), d(x,y), d(y,Ty)\}$$

For each $x, y \,\epsilon\, X, x \neq y, where\ \alpha, \beta, \gamma \geq 0, 1 + \alpha < \beta + \gamma$ then S &T have a common unique fixed point.

Proof: Let x_0 be arbitrary point of x. Define the sequence by $x_1 = Sx_0$ and $x_2 = Tx_1$

$$d(x_{2n+1}, x_{2n+2}) + \alpha d(x_{2n}, x_{2n+1}) \geq \frac{\beta d(x_{2n+1}, x_{2n+2})[1 + d(x_{2n}, x_{2n+1})}{1 + d(x_{2n}, x_{2n+1})} +$$

$$\gamma\, max[\,(x_{2n}, x_{2n+1}), d(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+2})]$$

$$d(x_{2n+1}, x_{2n+2}) + \alpha d(x_{2n}, x_{2n+1}) \geq \beta d(x_{2n+1}, x_{2n+2}) + \gamma d(x_{2n+1}, x_{2n+2})$$

$$\alpha d(x_{2n}, x_{2n+1}) \geq (\beta + \gamma - 1) d(x_{2n+1}, x_{2n+2})$$

$$d(x_{2n+1}, x_{2n+2}) \leq \frac{\alpha}{(\beta+\gamma-1)} d(x_{2n}, x_{2n+1})$$

In general

$$d(x_{n+1}, x_{n+2}) \leq \frac{\alpha}{(\beta+\gamma-1)} d(x_n, x_{n+1})$$

We proceed as follows

$$d(x_n, x_{n+1}) \leq \delta\, d(x_{n-1}, x_n)\ \ where\ \delta = \frac{\alpha}{(\beta + \gamma - 1)}$$

$$d(x_n, x_{n+1}) \leq \delta^n\, d(x_0, x_1)$$

Now we shall prove that $\{ x_n \}$ is a Cauchy sequence, for this we take a positive integer P, we have

$$d(x_n, x_{n+P}) \leq d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \cdots + d(x_{n+P-1}, x_{n+P})$$

$$\leq (\delta^n + \delta^{n+1} + \cdots + \delta^{n+P-1}) d(x_0, x_1)$$

$$\leq \delta^n / (1\text{-}\delta)\, d(x_0, x_1)$$

Since $0 \leq \delta < 1$,then $n \to \infty, \delta^n (1-\delta)^{-1} \to 0.$ *Hence* $d(x_n, x_m) \to 0$ *as* $m, n \to \infty$

it implies that $\{x_n\}$ is a Cauchy sequence in X, there exists a point

$z \epsilon X$ such that $x_n \to z$, then the subsequences $Sx_{2n} \to z$ and $Tx_{2n+1} \to z$.

Uniqueness: w is another fixed point of S & T

$$d(Sz, Tw) = d(z, w) \geq$$

$$-\alpha d(z, w) + \frac{\beta d(w, Tw)\,[1 + d(z, Sz)]}{1 + d(z, w)} + \gamma \max[d(z, Sz), d(z, w), d(w, Tw)]$$

$$d(z, w) \geq -\alpha d(z, w) + \gamma\, d(z, w)$$

$$d(z, w) \geq (\gamma - \alpha) d(z, w)$$

$$d(z, w) \leq \frac{1}{(\gamma - \alpha)}\, d(z, w)$$

$d(z, w) = 0$ [As $\gamma > \alpha$ and by prop [2.2]

$$z = w$$

This completes the proof of the theorem.

REFERENCES

[1] D. Ilic, V. Rakocevie, Quasi contraction on a cone metric space, Applied Mathematics Letters, Article in press.

[2] D. Turkoglu and M. Abuloha and T. Abdeljawad, KKM mapping in Cone metric spaces andsome fixed point theorems, Nonlinear Analysis: Theory, Methods and Applications, Vol.72, No.1,PP 348 -353,2010.

[3] F.E. Browder, W.V. Petryshyn,The solution by iteration of non linear functional equations in Banach spaces.Bull.Amer Math. Soc.,72,[1966],571-575.

[4] Hardy C.E. and Rogers T.D. A generalization of fixed point theorem of Reich canad ,Math. Bull.16,1973,201-206.

[5] I.Sahin and M. Telci, fixed point of contractive mappings on complete cone metric spaces,Hacettepe journal of mathematics, Vol.38, No.1,2009 page 59-67.

[6] J. o. olaleru, some generalizations of fixed point theorems in cone metric spaces,Fixed Point Theory and Applications;Vol.2009,Article ID 657914.

[7] Jungck G., compatible mappings and common fixed point,Inter. J. Math. Sci,Vol 9, (1986) 43-49,

[8] Jungck G. and Rhodes , B.E ,fixed points for set valued functions without continuity, Indian J. pure Appl. Math. Vol.29, No.3, 1998, 227-238.

[9] Kang, S.M., Fixed Point for Expansion Mapping,Math. Japonica 38(1993) 713-717.

[10] L.G. Huang and X.Zhang, cone metric spaces and fixed point theorem on contractive mappings, J. Math. Anal. Appl. Vol.332, No.2, 2007,PP 1468-1476.

[11] Park , S. and Rhoades , B.E.,Some fixed point theorems for expansion mapping, Math.Japon 33(1) (1988), 129-132.

[12] Pathak, H. K., Tiwari, R., Fixed point theorem for expansion mapping satisfying implicit relations, Filomat 20:1(2006) 43-57.

[13] Rajesh Kumar saini and Mohit Kumar ,common fixed point of Expansion Type Map in Cone Metric Space using Implicit Relation, Advances in Fixed point Theory2,Vol. No.3,2012,326-339.

[14] Rosenholtz Local Expansion derivatives and fixrd points,fund,Math 91 (1976)

[15] S. Chouhan and N. Malviya , A fixed point theorem for expansive type mapping in cone metric space , International Mathematical Forum, Vol.6, No.18,2011,PP 891-897.

[16] Shobha Jain, Shishir Jain, Lal Bahadur, compatibility and weak compatibility for four self maps in cone metric space. Bulletin of Mathematical Analysis and Applications,Vol.2,Issue 1, [2010] PP 15-24.

[17] Taniguchi,T., Common fixed point theorems on expansion type mapping on complete metric space, M.Japonica,34(1989),139-142.

[18] Wang, S.Z., Li, B.Y., Gao, Z.M.,Iseki, K., Some fixed point Theorems on expansion mappings, Math. Japon., 29(1984),631-636.